Problem 15.5

A particle's motion is described as:

$$x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$$

Note: The general expression for simple harmonic motion is:

$$x(t) = A\cos\left(\omega t + \phi_1\right) \quad \text{or} \quad x(t) = A\sin\left(\omega t + \phi_2\right)$$

where the parameters are:

"A" is the amplitude of the motion—this has the units of "meters;"

"ω" has the units of "radians/second" and is the *angular frequency* of the motion (this variable is frequency related and is sometimes written as $2\pi\nu$, where the "ν" is the motion's *frequency*);

"ϕ" is the *phase shift* and has the units of "radians"—its presence defines where the body is in its motion at *t = 0*.

These parameters will make more sense as you see them more in class. For now:

a.) With $x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$, what is the *frequency*?

Comparing:

$$x(t) = A\cos\left(\omega t + \phi_1\right)$$

and:

$$x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$$

we can see that:

$$\omega = 3.00\pi$$

But

$$\omega = 2\pi\nu$$

$$\Rightarrow \quad \nu = \frac{\omega}{2\pi}$$

$$= \frac{3.00\pi}{2\pi}$$

$$= 1.50 \text{ cycles/second} \quad (\text{or } \text{sec}^{-1} \text{ or Hz})$$

b.) With $x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$, what is the motion's *period*?

The relationship between the period and frequency is:

$$T = \frac{1}{\nu}$$

$$= \frac{1}{\left(1.50 \text{ cycles/second}\right)}$$

$$= .667 \text{ sec/cycle}$$

c.) What is the *amplitude*?

Comparing:

$$x(t) = A\cos\left(\omega t + \phi_1\right)$$

and:

$$x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$$

we get:

$$A = 4.00 \text{ m}$$

d.) What is the *phase constant*?

Comparing:

$$x(t) = A\cos\left(\omega t + \phi_1\right)$$

and:

$$x(t) = 4.00\cos\left(3.00\pi t + \pi\right)$$

we get:

$$\phi = \pi \text{ radians}$$

e.) Where is the body at t = .250 seconds?

$$x(t) = 4.00\cos\left(\left(3.00\pi \text{ rad/s}\right)\left(.250 \text{ s}\right) + \left(\pi \text{ rad}\right)\right)$$

$$= 2.83 \text{ m}$$